2015年度日本政府(文部科学省)奨学金留学生選考試験

QUALIFYING EXAMINATION FOR APPLICANTS FOR JAPANESE
GOVERNMENT (MONBUKAGAKUSHO) SCHOLARSHIPS 2015

学科試験　　問題

EXAMINATION QUESTIONS

(高等専門学校留学生)

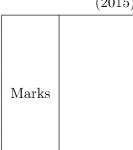COLLEGE OF TECHNOLOGY STUDENTS

数　　　学

MATHEMATICS

注意　☆試験時間は 60 分

PLEASE NOTE : THE TEST PERIOD IS 60 MINUTES.

Nationality		No.		Marks	
Name	(Please print full name, underlining family name)				

MATHEMATICS

1 Answer the following questions and write your answers in the boxes provided.

1) Solve the equation $x^3 + 4x^2 + 4x + 1 = 0$.

$$x =$$

2) Solve the equation $\cos 2x + \cos x = 0$ $(0 \le x \le 2\pi)$.

$$x =$$

3) Solve the inequality $3^{x+1} + \dfrac{1}{3^x} < 4$.

4) Solve the inequality $\log_2 \sqrt{2x - 1} < \log_4 x$.

5) Calculate $\displaystyle\sum_{n=0}^{120} \frac{1}{\sqrt{n+1} + \sqrt{n}}$.

6) How many integers n are there such that $1 \le n \le 200$ and n is not divisible by 2 nor 5 ?

7) Let $\vec{a} = (1,\ 3,\ -2)$, $\vec{b} = (2,\ 1,\ 1)$, $\vec{c} = (-3,\ 1 - s,\ t)$. Find the values of s and t such that $\vec{c}$ is perpendicular to $\vec{a}$ and $\vec{b}$.

$s =$ \qquad $t =$

8) Find the tangent line to the curve $y = e^{\frac{x}{2}}$ which goes through the point $(0,\ 0)$.

$y =$

9) The sequence $\{a_n\}$ satisfies the following conditions. Calculate $\displaystyle\sum_{n=1}^{\infty} a_n$.

$$a_1 = 5, \qquad a_{n+1} = \frac{3}{4}a_n \quad (n = 1,\ 2,\ 3,\ \cdots)$$

10) Calculate $\displaystyle\lim_{x \to 0} \frac{1 - \sqrt{1 - \sin x}}{x}$.

11) Let $f(x) = x - \log_e(2x + 1)$. Solve the inequality $f'(x) < 0$.

12) Calculate $\displaystyle\int_0^{\pi} x \sin x \, dx$.

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Let $A = a\begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}$ $(a > 0)$ and $I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ satisfy $A^4 + I = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$.

Answer the following questions and write your answers in the boxes provided.

1) Find a.

$$a = \frac{1}{\sqrt{2}}$$

2) Find the minimum positive integer n such that $A^n \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$.

$$n = 6$$

3) Find A^{2014}.

$$A^{2014} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

3   Let n be a positive integer. Answer the following questions and write your answers in the boxes provided.

1) Let $f(x) = x(1 - x)^n$. Solve the equation $f'(x) = 0$ in the open interval $0 < x < 1$.

$$x = $$

2) Let a_n be the maximum value of $f(x) = x(1 - x)^n$ in the closed interval $0 \leq x \leq 1$. Calculate $\lim\limits_{n \to \infty} (n + 1)a_n$.

3) Calculate $\displaystyle\int_0^1 x(1 - x)^n \, dx$.